

A new way to optimize LDPC code in gaussian multiple access channel

JINGXI ZHANG²

Abstract. The code design for m-user Gaussian Multiple Access Channel (GMAC) under fixed channel gains is studied. An approach to optimize the Degree Distribution Pair (DDP) of Low-Density Parity-Check (LDPC) code by means of Particle Swarm Optimization(PSO) in GMAC is proposed. The PDF of the log-likelihood-ratios(LLR) fed to the component LDPC decoders and channel node are derived, an LDPC-Coded-IDMA-PSO algorithm for GMAC are proposed. In the algorithm, particles carrying DDP move randomly to search the optimal. Compared with existing genetic algorithm, performance gain of 1.9dB can be obtained by simulation.

Key words. Ldpc, idma, gmac, ddp, pso.

1. Introduction

The prominent problem of Gaussian Multiple Access Channel (GMAC) is its Multiple Access Interference (MAI). Multiple User Detection (MUD) of CDMA system is impractical to implement [1]due to exponentially complexity. To overcome this issue, Interleaver-Division Multiple Access(IDMA) was proposed [2]. Owing to the capacity-achieving performance of Low-Density Parity-Check (LDPC) codes, they have received attention. Gaussian Approximation (GA) [3] was used to design Degree Distribution Pair (DDP) by getting the Probability Density Function (PDF) on Factor Graph (FG) [4].

However, the LDPC code constructed by the DDP of single user channel is of poor performance for GMAC due to the interference. Therefore, new design and optimization method for GMAC must be explored[5]. However, the approach has the drawback of trapping into local optimal [5]. LDPC code design for two-user GMAC was studied by Gaussian Mixture distributions[6]. Still it only considered two-user case. A low complexity density evolution analysis for two-user GMAC is considered [7]. However, the design of LDPC code for m-user remains unsolved.

¹Acknowledgment - The paper is supported by "the Fundamental Research Funds for the Central Universities (2016 MS100)". The author would like to thank the reviewers for their comments.

²Workshop 1 - Lecturer, Department of Electronic and Communication Engineering, North China Electric Power University, Baoding, Hebei, P.R.C ; e-mail: zhjxgeb@sohu.com

Zhang X. made an attempt to enhance the performance of IDMA system by designing LDPC DDPs [8]. However, existing LDPC code optimization in GMAC generally uses genetic algorithm, which has the problem of slow convergence and trapping into local optimal. Demodulation and decoding are jointly implemented to improve performance,. This inspired us to research the joint detection and decoding.

2. System Model

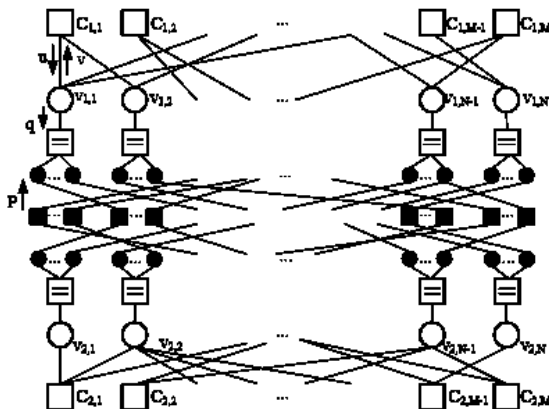


Fig. 1. Factor graph of ldpc-coded idma system

Figure 1 shows the FG of LDPC-Coded IDMA system. The hollow square and the circle denote Check Nodes (CN) and Variable Nodes (VN) respectively. The squares with equal denote spreading operation. Black squares denote Channel Factor Nodes (CFN) [2].

3. Density Evolution and Degree Distribution Pair Optimization

3.1. Density Evolution of Using Gaussian Approximation

The messages are expressed in LLR form. Let p, q, v, u denote the messages from CFN to VN, from VN to CFN, from VN to CN and from CN to VN respectively. All update is in extrinsic manner, i.e. a node gets its message from the neighbors of the node except the one that it sends to. The PDF of messages is Gaussian and consistent Gaussian [3]. To make the paper self-contained, the message transferring process is shown. Denotes $v_{i,n}$ by a VN which connects to i CNs and n CFNs, the means of output messages of $v_{i,n}$ at the l -th iteration $\bar{v}_{i,n}^{(l)}$ can be expressed as

$$\bar{v}_{i,n}^{(l)} = n \cdot \bar{p} + (i - 1) \cdot \bar{u}^{(l-1)} \tag{1}$$

Where \bar{p} denote the means of output message of CFN to VN, and $\bar{u}^{(l-1)}$ denote

the means of output message of CN to VN at the $(l-1)$ -th iteration. The expectation of u can be computed by weighting the means of CN with $\{\rho_j, 2 \leq j \leq d_c\}$, then summing.

$$\overline{u^{(l)}} = \sum_{j=2}^{d_c} \rho_j \phi^{-1} \left(1 - \left(1 - \sum_{i=2}^{d_v} \lambda_i \cdot \phi \left(\overline{v_{i,n}^{(l)}} \right) \right)^{j-1} \right) \quad (2)$$

Where $\phi(x)$ is defined in [3]. The outgoing messages to a CN have the mixture density

$$f_v^{(l)} = \sum_{i=2}^{d_v} \lambda_i N \left(\overline{v_{i,n}^{(l)}}, 2\overline{v_{i,n}^{(l)}} \right) \quad (3)$$

The messages from VN at the l -th iteration can be got by weighting $\overline{v_{i,n}^{(l)}}$ with λ_i and summing

$$\overline{v^{(l)}} = \sum_{i=2}^{d_v} \lambda_i \overline{v_{i,n}^{(l)}} \quad (4)$$

The decoded bit is used to refine the ESE in the iteration. And the message from VN to CFN is

$$q = \sum_{i=1}^{d_s-1} p_i + \sum_{i=1}^{d_c} u_i \quad (5)$$

Denote the means of the output message from $v_{i,n}$ at the l -th iteration to CFN by $\overline{q_{i,n}^{(l)}}$

$$\overline{q_{i,n}^{(l)}} = (n-1) \cdot \overline{p} + i \cdot \overline{u^{(l)}} \quad (6)$$

Weighting $\overline{q_{i,n}^{(l)}}$ with coefficient λ_i , the output message to CFN from VN at the l -th iteration is

$$\overline{q^{(l)}} = \sum_{i=2}^{d_v} \lambda_i \overline{q_{i,n}^{(l)}} \quad (7)$$

The outgoing message to CFN has the following mixture density

$$f_q^{(l)} = \sum_{i=2}^{d_v} \lambda_i N \left(\overline{q_{i,n}^{(l)}}, 2\overline{q_{i,n}^{(l)}} \right) \quad (8)$$

The CFNs perform ESE and the output LLR of a given chip can be expressed as

$$\overline{p^{(l)}} \approx \frac{2 \cdot h_k}{\sigma^2 + (K-1) \sum_{i=2}^{dv} \lambda_i \varphi \left(\overline{q_{i,n}^{(l-1)}} \right)} \quad (9)$$

Where K is the user number, and $\overline{q_{i,n}^{(l-1)}}$ is the means of message from $v_{i,n}$ to CFN at the $(l-1)$ -th iteration, and h_k is the channel coefficient of the k -th user. For simplicity we assume that all h_k are 1. To compute $\overline{p^{(l)}}$ efficiently, a function is defined:

$$\varphi(x) = \frac{1}{\sqrt{4\pi x}} \int_{-\infty}^{+\infty} \left(1 - \tanh^2 \left(\frac{u}{2} \right) \right) \exp \left(-\frac{(u-x)^2}{4x} \right) du \quad (10)$$

Therefore $\overline{p^{(l)}}$ can be approximated as:

$$\overline{p^{(l)}} \approx \frac{2 \cdot h_k}{\sigma^2 + (K-1) \sum_{i=2}^{dv} \lambda_i \varphi \left(\overline{q_{i,n}^{(l-1)}} \right)} \quad (11)$$

3.2. Iterative Procedure

After initialization procedure, iterative decoding is performed by passing LLR messages between the CN, VN and CFN. One complete iteration starts at updating $\overline{p^{(l)}}$ using (9), then updating $\overline{v^{(l)}}$ using (4), and then updating $\overline{u^{(l)}}$ using (2), finally updating $\overline{q^{(l)}}$ using (6). After a certain number of iteration $G_{Aiter_{\max}}$, mean Bit Error Rate (BER) can be calculated using (3).

3.3. Mathematical model

The mathematical model can be expressed as (12).

$$\begin{cases} \min & P_e \\ \text{s.t.} & \sum_{i=2}^{d_c} \frac{\rho_i}{i} / \sum_{i=2}^{d_v} \frac{\lambda_i}{i} = 1 - R \end{cases} \quad (12)$$

Where P_e the mean BER of the optimized DDP, R is the code rate of LDPC code.

4. Proposed LDPC-Coded-IDMA- PSO algorithm

This paper proposes a LDPC-Coded-IDMA-PSO algorithm for searching the optimal or near optimal DDP. The schematic flow chart of the algorithm is shown in Figure 2.

Step 1) Initialization

$$\sigma = \sigma_{\min}, \quad \overline{p^{(0)}} = 2 / (\sigma^2 + (K-1)), \quad \overline{v^{(0)}} = 0, \quad \overline{u^{(0)}} = 0, \quad \overline{q^{(0)}} = 0, \quad p_g^{\min} = 0, \quad p_g^{\max} = 1$$

This procedure sets the initial value of parameters used in GA and PSO, where p_g^{\min} and p_g^{\max} specify the lower and upper bounds of particle, respectively.

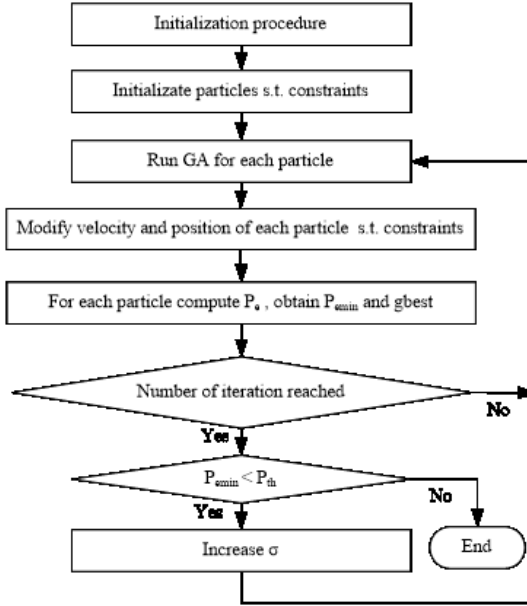


Fig. 2. The schematic flow chart of ldpc-coded-idma-pso algorithm

Step 2) Initialize particles s.t. the constraint in (12). Each particle contains the DDPs code.

Step 3) for each particle run GA algorithm for n_{GA} times.

Step 4) modify each particle according to (13) and (14), respectively, s.t. the constraint in (12).

$$v_{i,d} = w \cdot v_{i,d} + c_1 \cdot rand_1() \cdot (p_{i,d} - x_{i,d}) + c_2 \cdot rand_2() \cdot (p_{g,d} - x_{i,d}) \quad (13)$$

$$\begin{cases} x_{i,d} = x_{i,d} + v_{i,d} \\ p_g^{\min} \leq x_{i,d} \leq p_g^{\max} \end{cases} \quad (14)$$

Where w is the inertia weight, it is set based on (15) and decreases from about 0.9 to 0.4 in a run.

$$w = (w_{\max} - w_{\min}) \left(1 - \frac{iter}{iter_{\max}} \right) \times iter + w_{\min} \quad (15)$$

The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward $pbest$ and $gbest$ positions. v_{id} is the velocity of a particle, $V_g^{\min} \leq v_{id} \leq V_g^{\max}$. p_{id} is $pbest$ of particle i , p_{gd} is the $gbest$ of the group.

If $v_{i,d} > V_g^{\max}$, then $v_{i,d} = V_g^{\max}$; If $v_{i,d} < V_g^{\min}$, then $v_{i,d} = V_g^{\min}$.

Step 5) For each particle, compute P_e , obtain $P_{e\min}$, the best DDP, and $gbest$ achieved so far. P_e is the cost (fitness) function, and calculated by (3).

Step 6) Judge whether the number of iteration reached, if not reached, continues PSO.

Step 7) Judge whether P_e is less than the error threshold P_{th} . If yes, increase the standard deviation of noise (σ) by $\Delta\sigma$, otherwise end the process and record the best DDP and σ .

Three types of iteration are present. The first is the LDPC code iteration, the second is CFN iteration, and the third is PSO iteration. The no are expressed as $n_{LDPC}, n_{CFN}, n_{PSO}$, respectively.

5. Numerical Results

We search for good DDPs by using LDPC-Coded-IDMA-PSO algorithm. Furthermore we analyze the performance of optimized DDP. R was set to 0.5, the Spreading Factor (SF) is half the number of users. In this way the spectrum efficiency of the system is set to 1. Assuming all the users have equal power and equal rate, it is unnecessary to make the DDP different between different users. In the LDPC-Coded-IDMA-PSO algorithm, the parameter is set as Table 1.

Table 1. The parameters of PSO process

c1	c2	w _{min}	w _{max}	R	d_v	SF
1	1	4	9	0.5	8,16,20	2

The LDPC code convergence threshold phenomenon in single user system can also be seen in MAC scenario. If the channel's noise standard variance is greater than the threshold, the error probability cannot approach zero. The result of the global fitness value is illustrated in Table 2.

Table 2. Global fitness value of different σ

σ	1.60	1.50	1.40	1.30
P_e	2.6165e-001	2.4449e-001	2.4054e-001	3.0867e-009

By running the LDPC-Coded-IDMA-PSO algorithm with $n_{LDPC} = 10$, $n_{CFN} = 20$, $n_{PSO} = 50$, and particle number $n_P = 10000$, we obtained the DDPs for maximal variable node degree $d_v = 8, 16, 20$. Table 3 shows the optimized DDPs, and the fitness value of the given the noise threshold σ^* for 4-user with $R=1/2$ and spectral efficiency 1.

Table 3. Optimized result by LDPC-Coded-IDMA-PSO algorithm

d_v	8	16	20
λ_2	0.468588	0.516962	0.727932
λ_3	0.250980	0.193965	0.043185
λ_4	0.023820	0.149168	0.088376
λ_5	0.231879	0.104677	0.099782
λ_8	0.024733		
λ_{16}		0.035227	
λ_{20}			0.040725
ρ_4			0.224478
ρ_5	0.600646	0.753479	0.775522
ρ_6	0.399354	0.246521	
The global fitness	7.720827e-8	4.927803e-8	3.086720e-9
The max Noise sigma	1.30	1.30	1.30

It can be seen for 4-user case, the bigger the maximum VN degree, the better the global fitness.

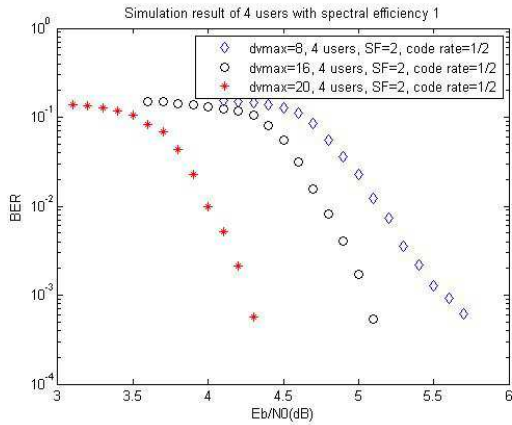


Fig. 3. Simulation results of 4 users with obtained DDPs

Simulation was carried out based on the DDPs from Table 3. In the simulation process, the user number was 4, Spreading Factor (SF) was 2, $R=1/2$, therefore the spectral efficiency was 1. $n_{LDPC}=10, n_{CFN}=20$ and code length is 4000. The simulation result is illustrated in Figure 3.

The results show compliance with that of DDPs in Table 3. The smaller the global fitness value in Table 3, the better the BER performance is under the same simulation condition.

To show the power of the proposed algorithm, a comparison was made between

the DDP obtained by genetic algorithm [9] and the DDP obtained by LDPC-Coded-IDMA-PSO algorithm. The DDP obtained by genetic algorithm is

$$\begin{cases} \lambda(x) = 0.262934 + 0.277661 \cdot x^2 + 0.110575 \cdot x^3 + 0.188802 \cdot x^4 \\ \rho(x) = 0.261626 \cdot x^5 + 0.738374 \cdot x^6 \end{cases}$$

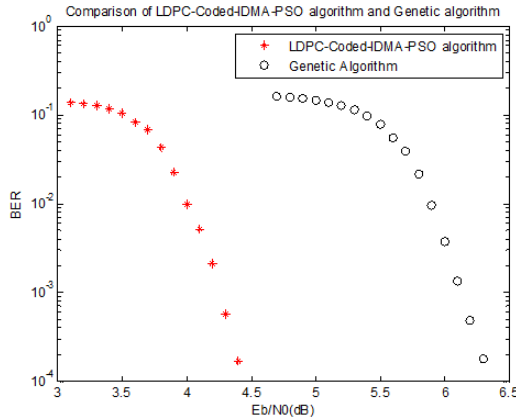


Fig. 4. Comparison of LDPC-Coded-IDMA-PSO algorithm and genetic algorithm

In the simulations, the condition as in Figure 3. The two methods have the same complexity and the result is shown in Figure 4. It can be seen the superiority of LDPC-Coded-IDMA-PSO algorithm is evident. At $BER = 10^{-4}$, the performance gain of 1.9dB can be achieved.

6. Conclusions

An algorithm for optimization the DDP of LDPC codes over GMAC was proposed to suppress Multiple Access Interference (MAI). Density evolution of LDPC coded IDMA system and the extrinsic information transfer process was derived. In the proposed LDPC-Coded-IDMA-PSO algorithm, particles carrying DDP move randomly to search for the optimal.

Convergence threshold phenomenon was observed. The greater the maximum VN degree, the less the mean BER. Simulation results showed the convergence of the proposed algorithm.

Compared with genetic algorithm, the algorithm has the merit of searching more effectively and avoid trapping into local optimal. Performance gain of 1.9dB was obtained by using algorithm.

References

- [1] M. ZUBAIR, A. N. MALIK: *Multuser detection for Asynchronous Multicarrier CDMA*

- using Particle Swarm Optimization*. IEICE TRANSACTIONS on Communications 91 (2008), No. 5, 1636–1639.
- [2] P. L. LIU, K. WU, W. K. LEUNG: *Interleave-Division Multiple-Access*. IEEE trans. wireless comm 5 (2006), No. 4.
 - [3] S. Y. CHUNG: *Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation*. IEEE Trans. Inform. Theory 47 (2001), No. 2, 657–670.
 - [4] F. R. KSCHISCHANG, S. K. JAIN: *Factor graphs and the sum-product algorithm*. IEEE Trans. Inf. Theory 47 (2001), No. 2, 498–519.
 - [5] M. N. GAIKWAD, K. C. DESHMUKH: *SC-LDPC Code With Nonuniform Degree Distribution Optimized by Using Genetic Algorithm*. IEEE Communications Letters 29 (2016), No. 9, 797–804.
 - [6] S. CHAKRAVERTY, R. JINDAL, V. K. AGARWAL: *LDPC Code Design for the Two-User Gaussian Multiple Access Channel*. IEEE Transactions on Wireless Communications 12 (2005) 521–528.
 - [7] N. L. KHOBRADE, K. C. DESHMUKH: *SC-LDPC code design for Gaussian multiple access channel*. IET Communications 30 (2005), No. 4, 555–563.
 - [8] Y. F. ZHOU, Z. M. WANG: *Boosts to LDPC-coded IDMA systems*. Electronics Letters 316 (2008), Nos. 1–5, 198–210.

Received November 16, 2016

